Kirchhoff's Rules

I. Theory

This experiment seeks to determine if the currents and voltage drops in a "two-loop" circuit obey Kirchhoff's rules. A two-loop circuit is a circuit that has two distinct paths through which current can flow. The currents and voltage drops in such a circuit containing multiple resistors and power supplies will be measured. A diagram of the two-loop circuit we will study today is shown in Figure 1 below.

One can use a simple algebraic method to calculate the voltage drop across and the current through each resistor. The algebraic method involves the application of Kirchhoff's two rules.

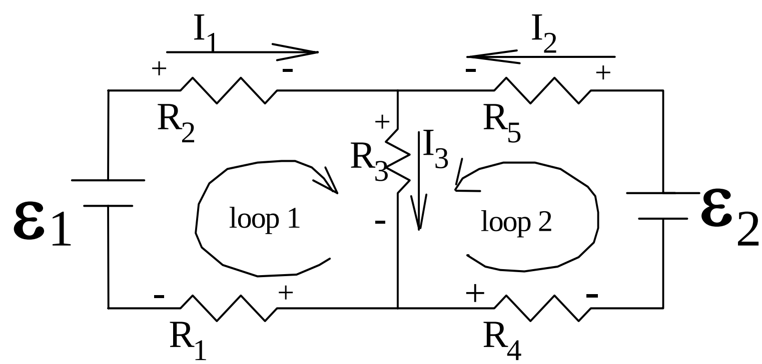

Figure 1. Two-loop circuit.

Loop Rule:
When any closed circuit loop is traversed, the sum of the voltage drops and rises around that closed loop must equal zero.

Junction Rule:
At any junction point in a circuit where the current can divide (such as where two or more wires connect), the sum of the currents into the junction must equal the sum of the currents out of the junction.

You will be able to test these rules directly, by measuring the currents and voltage drops involved. The procedure below explains how to do this. But Kirchhoff's rules can also be used to predict the exact values of the currents flowing in the circuit of figure 1. We will show you the calculation here, and leave it to you to compare the predictions with experimental measurements.

When we analyze the circuit shown above using Kirchhoff's rules we obtain the following three equations:

$$(a)\quad \varepsilon_1 = \Delta V_2 + \Delta V_3 + \Delta V_1 \qquad \text{loop 1}$$
$$(b)\quad \varepsilon_2 = \Delta V_5 + \Delta V_3 + \Delta V_4 \qquad \text{loop 2}$$
$$(c)\quad I_3 = I_1 + I_2 \qquad \text{either node}$$

Here ΔV represents the voltage drop across one of the resistors, from plus to minus as indicated on the circuit diagram.

We can us Ohm's law, $V=IR$, to rewrite equations (a) and (b) in terms of I_1 and I_2 and then solve the three equations for I_1 and I_2. This gives the following result:

(1) $I_2 = (1/R_3)(\varepsilon_1 - I_1(R_1+R_2+R_3))$

(2) $I_1\{(1/R_3)(R_1+R_2+R_3)(R_3+R_4+R_5)-R_3\} = \varepsilon_1\{(1/R_3)(R_3+R_4+R_5)\}-\varepsilon_2$

In the first circuit you will study, all the resistors will have the value of approximately 100 Ω, and the power supplies ε_1 and ε_2 will be set at approximately 10 V and 5 V, respectively. Substituting in these values gives, for the three currents,

I_1 = 31.25 mA

I_2 = 6.25 mA

I_3 = 37.5 mA

Use these values as theoretical predictions, to compare with measurements which you will make.

II. Experimental Procedure

Important Note: In the following procedures, the initial meter's scale settings are given as a general starting place for you to begin making your measurements. You should, however, use a meter scale, according to the value being measured, so that you obtain the most precise reading possible. For example, if you are measuring 1 mA, a full-scale setting of 4 mA gives a much more precise measurement than a full-scale setting of 400 mA. Newer meters such as ours may "autorange" to the most sensitive setting possible.

A. Verifying Kirchhoff's Rules

The circuit should be pre-connected with the 100Ω resistors (on black two-pin plugs) as shown Figure 1, on a plug-in circuit board. (See Appendix A for details on the circuit board.) Assume the resistors on the circuit board are all exactly 100Ω. In this part you will use the values of the currents calculated above, using equations (1), (2), and (c) and assuming the resistors are exactly 100 Ω. Then you will actually measure the currents and from your measurements decide whether or not the equations you used to calculate the theoretical values are accurate. Record all measurements in a table in your lab book.

Q1. Which of the five resistors should carry the same current and which currents, in terms of I_1, I_2, and I_3, are they?

1. We will be using two power sources in this experiment. Adjust one to **exactly** 10.00 V; this is ε_1.
2. Adjust the other power supply to **exactly** 5.00 V; this is ε_2.
3. Connect the power supplies to the circuit as shown in Figure 1. (See Appendix A.)
4. Turn the ammeter to the "400mA" setting. Measure the current I_1 by measuring the current through resistor R_2. (Remove the gray jumper cable near R_2 and connect the ammeter leads where the jumper cable used to go.) Record I_1. Remove the ammeter leads and replace the gray jumper cable.
5. Measure the current I_2 by measuring the current through resistor R_5. Record I_2. Follow a procedure similar to that used in step (4), but removing the jumper next to R_5.
6. Measure the current I_3 by measuring the current through resistor R_3. Record I_3.
7. Also measure and record the currents through resistors R_1 and R_4.
8. Turn the multimeter to the "V" setting. (Be **sure** to do this.) Measure the voltage drop across each resistor. Record ΔV_1, ΔV_2, ΔV_3, ΔV_4, and ΔV_5.

Q2. Do your measurements support your answer to Q1? Why or why not?

Q3. Compare the values for I_1, I_2, and I_3 that you measured to the values that were calculated using equations (1), (2), and (c). Do the calculated values agree closely enough with the measured values to trust that equations (1), (2), and (c) are correct expressions for the currents?

Q4. Using your measured values, does $I_3 = I_1 + I_2$? Can you assert with confidence that the measured currents obey Kirchhoff's Junction Rule?

Q5. Using your measured values, are equations (a) and (b) valid, within reason? Explain.

B. Detailed Measurements on a Two-Loop Circuit

The two-loop circuit in this part of the experiment will be constructed using approximately 75 Ω resistors (some other value may possibly be supplied). In this section, once again, you will measure the currents and voltage drops, and from your measurements you will decide if the currents and voltage drops in the two-loop circuit obey Kirchhoff's rules. Also, you will determine if the labeling scheme for the currents used in your circuit diagram is consistent with the values measured during the experiment. Please make a table of your measurements.

1. Turn off the power supply. Remove the five black two-pin resistor plugs from the circuit board.
2. Measure the resistance of each 75 Ω resistor (on red 2-pin plugs) before you plug it in, and record in your lab book. I suggest that you begin constructing the circuit at R_1. Measure the resistance of R_1 and then plug in that resistor plug to the circuit board in the R_1 position according to the circuit diagram. Continue with the other red plugs until all five are in.
3. Connect the two power supplies to the circuit as shown in figure 1 if not already done.
4. Turn the ammeter to the "400mA" setting. Turn on the power supply and measure the currents through the five resistors and record in your lab book.
5. Turn the voltmeter to the "V" setting. Measure the voltage difference provided by each power supply and the voltage drop across each resistor.

Q6. Do your measurements support your answer to Q1? Why or why not? Explain your answer in detail, using the experimental values as proof for your answer.

Q7. Explain in detail if $I_3 = I_1 + I_2$ for your <u>complete</u> set of measured current values (at *both* of the junctions in the circuit). How does this relate to conservation of electric charge?

Q8. Discuss, using your experimental data as support for your answer, whether or not the equations (a) and (b) are satisfied, within reasonable margins. Explain.

Q9. Why are the voltages in Part B the same as in Part A, while the currents are different? A qualitative argument will be good enough.

C. Cleanup. Please replace all red resistor plugs with black ones. Place the reds in storage holes.

III. Equipment

Kirchhoff Board
Five 100 Ω resistors mounted on black two-pin plugs
Five 75 Ω resistors mounted on red two-pin plugs
Multimeter (Metex M-3850)
Dual power supply with +5V and 0-15 V (HY30030-3)
Banana cords: 5 short gray, three long black, and three long red.

IV. Appendix A: The Kirchhoff Circuit Board

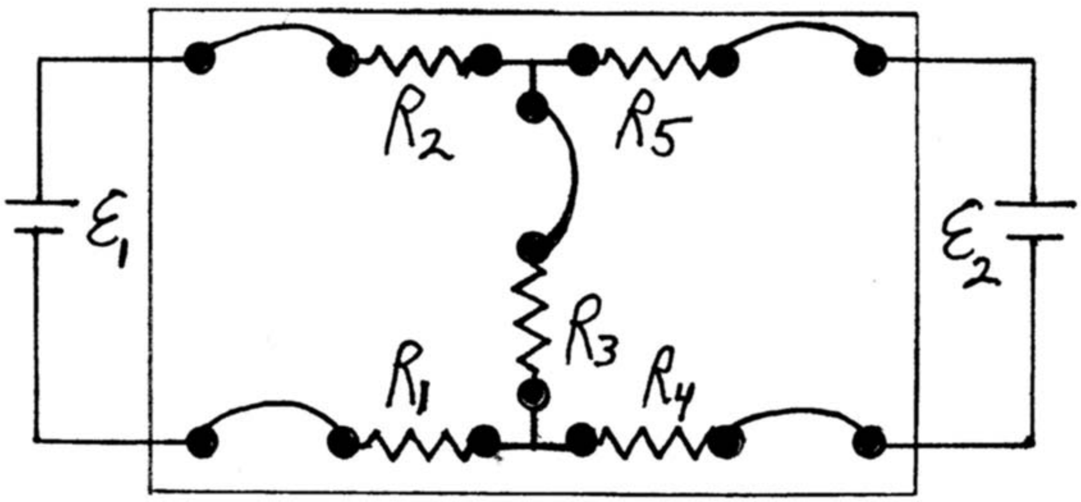

Fig. A-1. Hookup diagram for Kirchhoff circuit board. The curved lines represent the gray jumper cables which can be replaced with ammeter connections to measure the current through the adjacent resistor. The resistors are mounted in two-pin plugs. The voltage sources ε_1 and ε_2 are located in the power supply unit.