Problem Set 8

Problem 1.  a. Find the bound-state energy of a particle in a finite square-well potential

\[ V(x) = \begin{cases} 
0 & x < -a \\
-g/2a & -a < x < a \\
0 & x > a 
\end{cases} \]

in the limit \( a \to 0 \).

b. Calculate the reflection and transmission coefficients in the same limit for an incident particle of momentum \( p \).

Problem 2. Find the bound-state energy, and the reflection/transmission coefficients, for a particle in the potential

\[ V(x) = -g \delta(x). \]

Hint: First show that

\[ \left. \frac{d\psi}{dx} \right|_{0+} - \left. \frac{d\psi}{dx} \right|_{0-} = -\frac{2mg}{\hbar^2} \psi(0) \]

(i.e., the first derivative is not continuous at \( x = 0 \)).

Second hint: To do this, integrate \( \frac{d^2\psi}{dx^2} \) from just on one side of the origin to the other, that is, from \( x = 0 - \varepsilon \) to \( x = 0 + \varepsilon \); and use the Schrodinger equation to relate this result to the potential energy \( V(x) \).