Problem Set 4

Problem 1. Suppose that at time $t = 0$ the state of a particle moving in one dimension is given by the wave-function

$$\psi(x) = Ne^{-|x-x_0|/2a}.$$ 

(a) What is the normalization constant $N$?
(b) Compute $\langle x \rangle$ and $\Delta x$.

Problem 2. Prove that:

(a) $\delta(cx) = \frac{1}{|c|} \delta(x)$
(b) $f(x) \frac{d^2}{dx^2} \delta(x) = \frac{d^2}{dx^2} f(x) \delta(x)$
(c) $f(x) \delta(g(x)) = f(x) \left| \frac{dg}{dx} \right|^{-1} \delta(x-x_0)$ where $g(x) = 0$ only at $x = x_0$.

Problem 3. Invent your own personalized delta function! That is, come up with a sequence of functions $\delta_L(x-y)$ that depend on a parameter $L$, which have the property

$$f(x) = \lim_{L \to \infty} \int_{-\infty}^{\infty} dy \delta_L(x-y) f(y)$$

What you are looking for is a curve which depends on $L$, such that: (i) for any $L$, the area under the curve is 1; and (ii) as $L \to \infty$, the area is peaked around the point $x = y$. 