Homework 8 due in class 12/5

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name Signature:

1. (15 points) Binney & Tremaine 1.2

2. (15 points) Binney & Tremaine 1.3

3. (10 points) Binney & Tremaine 2.1. Note: The statement “in which the density satisfies \( \lim_{r \to 0} \rho r^{5/2} \)” is an aside; it is simply meant to reassure you that you can assume the density profile will not be steeper than is physically possible. It is not a statement about the specific choice of density profile and you should solve the problem for general density profiles.

4. (20 points) Prove that it is impossible to have a spherical distribution of matter in which the mass density is everywhere a constant times the gravitational potential, unless that constant is zero.

5. (10 points) Binney & Tremaine 2.8

6. Consider the following two spherical mass profiles. The first provides a good description of the total gravitating mass in globular clusters, whereas the other describes large elliptical galaxies. The mass profile extends to infinity.

\[
M_H(r) = \frac{M_0 r^3}{(b^2 + r^2)^{3/2}} \quad \text{Plummer profile}
\]

\[
M_P(r) = \frac{M_0 r^2}{(b + r)^2} \quad \text{Hernquist profile}
\]

For each profile:

(a) (5 points) What is the total mass?

(b) (5 points) What is the density profile? Draw or plot the two on top of each other on a log-log scale.

(c) (5 points) What is the gravitational potential? State your chosen zero point for the potential. What is the gravitational potential when \( b = 0 \)?

(d) (5 points) Find and sketch the circular velocity profile. What is the peak circular velocity?

(e) (5 points) Find the total gravitational potential energy.

(f) (5 points) How are the two models similar? How do you explain the difference between their total gravitational potential energies? What else do you see as being the biggest difference between the two models? Why?