Homework 7 due 5:10PM 11/3

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name: Signature:

1. (30 points) This problem deals with alternative formulations of Lagrangian mechanics.

   (a) (15 points) Imagine that we hadn’t heard of the Euler-Lagrange equations, and we only knew Hamilton’s equations. Derive the Euler-Lagrange equations directly from Hamilton’s equations. 
   
   *Hint*: Start by writing the Lagrangian in terms of the Hamiltonian (i.e., turn around the definition of the Hamiltonian to solve for the Lagrangian), and use $p_i = \partial L / \partial \dot{q}_i$.

   (b) (15 points) Now imagine we came up with an alternative Lagrangian:

   $$L' (\dot{p}, p, t) = -\dot{p}_i q_i - H (q, p, t)$$

   Find the analogue of the Euler-Lagrange equations in the ($\dot{p}, p$) system.

2. (40 points) Start from the $(x, y, z)$ ($p_x, p_y, p_z$) coordinate system. These are the “old” coordinates. Define $L = r \times p$. First show that if we transform to a coordinate system where two of the new momenta are components of the angular momentum (e.g. $P_1 = L_x$ and $P_2 = L_y$), then the transformation is not canonical. But show that if we use the square of the total angular momentum together with another component of the angular momentum, (e.g. $P_1 = L^2$ and $P_2 = L_z$) then the transformation is canonical. What should we use for $P_3$? Does this remind you of anything in quantum mechanics?

3. (30 points) Show using any method of your choosing that the transformation

   $$Q = \log \left( \frac{\sin p}{q} \right) ; \ P = q \cot p$$

   is canonical.