1. (30 points) This problem deals with alternative formulations of Lagrangian mechanics.

   (a) (15 points) Imagine that we hadn’t heard of the Euler-Lagrange equations, and we only knew Hamilton’s equations. Derive the Euler-Lagrange equations directly from Hamilton’s equations. 

       Hint: Start by writing the Lagrangian in terms of the Hamiltonian (i.e., turn around the definition of the Hamiltonian to solve for the Lagrangian), and use \( p_i = \frac{\partial L}{\partial \dot{q}_i} \).

   (b) (15 points) Now imagine we came up with an alternative Lagrangian:

       \[ L'(p, \dot{p}, t) = -\dot{p}_i q_i - H(q, p, t) \]

       Find the analogue of the Euler-Lagrange equations in the \((\dot{p}, p)\) system.

2. (40 points) Start from the \((x, y, z) (p_x, p_y, p_z)\) coordinate system. These are the “old” coordinates. Define \( L = r \times p \). First show that if we transform to a coordinate system where two of the new momenta are components of the angular momentum (e.g. \( P_1 = L_x \) and \( P_2 = L_y \)), then the transformation is not canonical. But show that if we use the square of the total angular momentum together with another component of the angular momentum, (e.g. \( P_1 = L^2 \) and \( P_2 = L_z \)) then the transformation is canonical. What should we use for \( P_3 \)? Does this remind you of anything in quantum mechanics?

3. (30 points) Show using any method of your choosing that the transformation

       \[ Q = \log \left( \frac{\sin p}{q} \right); \ P = q \cot p \]

       is canonical.