Physics 701: Classical Mechanics
San Francisco State University ©2017 Andisheh Mahdavi Fall 2017, TuTh 5:10PM

Homework 4 Due 5:10PM Thursday 10/12

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences.

Name ______________________ Signature: ______________________

1. Assume that initially the planets conduct circular orbits around the Sun. The potential is

\[ U(r) = G_n M_\odot m_P r^n / n \]

where \( n = -1 \), \( G_{-1} \) is the Newtonian Gravity constant, \( M_\odot \) is the sun’s mass, and \( m_P \) is the planet’s mass.

Imagine that a godlike being is able to instantaneously change the law of gravity so that \( n \) takes on some other value between -4 and 0. This being is benevolent, and chooses at the same time to change \( G_n \) such that the Earth’s orbit is in no way disturbed—this means that the path of the orbit stays the same and there is no sudden “jerk” to the Earth’s motion. Note that the units of \( G \) have to change when \( n \) changes so the potential can continue having units of energy.

When \( n \) is changed, the motions of all the planets other than the Earth change, and this problem tries to get you to think through what the orbits now look like in this new reality. While Mathematica is required to answer the second part of this problem, this is not considered a separately graded Mathematica project. Look at the class web page for an example of numerical root finding.

- (5 points) Argue using words and not equations that angular momentum should be conserved when \( n \) and \( G_n \) are varied, but not energy.
- (20 points) Calculate the bounds on \( n \) so that all the major planets stay within the solar system (i.e., do not escape) and also do not crash into the Sun.
- (10 points) Calculate the allowed range of \( n \) for which Jupiter does not cross Earth’s orbit.

2. (10 points) Show explicitly that Landau equation 15.5 is the equation for an ellipse satisfying

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

3. (15 points) Come up with an \( r(\phi) \) which cannot under any circumstance conserve angular momentum and therefore cannot correspond to a two-body orbit in a central force potential.

4. Consider two bodies in the central force potential:

\[ V(r) = E_0 \frac{r}{r + r_0} \]

where \( E_0 \) and \( r_0 \) are two positive constants with units of energy and distance respectively. These types of potentials are of great physical interest in isothermal stellar systems. While Mathematica is required to answer the latter parts of this problem, this is not considered a separately graded Mathematica project.
(a) (2 points) Is escape possible?

(b) (5 points) What is the relationship between the radius and velocity of a circular orbit?

(c) (5 points) What is the relationship between the radius and energy of a circular orbit?

(d) (3 points) What is the energy of a circular orbit at $r_0$?

(e) (7 points) Write down the Euler-Lagrange equations for this system in polar coordinates.

(f) (10 points) Use Mathematica to solve the second-order E-L equations numerically. Plot the trajectories of one bound orbit for at least 10 periods.

(g) (8 points) Imagine that the two bodies never get closer than $r_0$ or farther than $2r_0$. Find their energy and angular momentum.