Physics 701: Classical Mechanics
Thornton Hall 425, San Francisco State University
Fall 2016, Tu-Th 5:10PM

Homework 1 Due 5:10PM Thursday 9/1

While I may have consulted with other students in the class regarding this homework, the solutions presented here are my own work. I understand that to get full credit, I have to show all the steps necessary to arrive at the answer, and unless it is obvious, explain my reasoning using diagrams and/or complete sentences. Please attach this sheet to your homework.

Name ___________________________ Signature: ___________________________

This assignment does not contain a Mathematica project, but it uses Mathematica as a plotting tool.

1. (15 points) A particle of mass \( m \) moving in the \( x \) direction sees an attractive potential energy function of the form \( V(x) = k|x| \) where \( k > 0 \). If it is released from rest with amplitude \( x_0 \), what is its velocity when it passes the origin? Find the period, by reduction to quadrature, for a single complete cycle of oscillation.

2. (50 points) In science fiction, spaceships or space stations frequently rotate to maintain artificial gravity. Imagine a hollow cylinder of radius \( R \), rotating uniformly with period \( P \) to produce artificial gravity equal to Earth’s gravity. Its inhabitants live on the inside the curved part of the cylinder in a protected environment. This problem is intended to help you design such a space station so that gravity ”feels” normal.
   - (10 points) **How fast it needs to spin.** Calculate the period \( P \) of the rotation required to produce Earth-type gravity as a function of the cylindrical radius \( R \).
   - (10 points) **Dizziness** Assume that your inner ear will feel dizzy if its speed differs from that of your feet by more than 10 cm/s. What are the bounds on the cylindrical radius \( R \) of the space station so that you don’t feel dizzy while standing still?
   - (10 points) **Cost.** Estimate the cost, in 2016 dollars, of building the cheapest space station that conforms to the above and can support life. Cite online sources but assume that with Space-X style reusable rockets, we can cut the cost of each launch from earth to $5,000,000. You will get graded on the soundness of your assumptions and not on the final number.
   - (10 points) **What happens if you drop something.** Imagine you live on this space station. In your own rest frame, you stand at \( x = 0 \), where positive \( x \) indicates the instantaneous direction of rotation. You drop something from a height \( y = h \). Which side of you \((x > 0 \text{ or } x < 0)\) does the object wind up? Write down the equation for its trajectory in the \( x,y \) plane as viewed by yourself in your rest frame. At what \( x \) values does it hit the ground? Use Mathematica to plot the trajectory for your “cheapest” space station. Hint: use \( \sin \theta \approx \theta \) for \( \theta << 1 \) to simplify the algebra.
   - (10 points) **What happens if you jump** Imagine that on Earth you can jump to a height of 1m. You and your friend both stand at \( x = 0 \) and you jump up in this space station while your friend watches. Which side of your friend \((x > 0 \text{ or } x < 0)\) do you wind up? Calculate the generic trajectory and use Mathematica to plot it for your “cheapest” space station.

3. (35 points) A new planet around Proxima centauri was just announced by ESO today. Breakthrough Starshot, which has Mark Zuckerberg and Stephen Hawking on its board, intends to send a lightweight probe there in the next couple of decades. The probe will be accelerated by infrared laser to nontrivial fractions of the speed of light. Neglecting the (small) relativistic effects, calculate the sustained power output required to accelerate a 5-gram probe such that it can get to Proxima centauri in 25 years.